RLM 31 ; 16 Jan 2013

NEO Apophis (99942) distance from parallax

NEO asteroid Apophis [99942] was observed at Rigel and SSO observatories on Jan 13, 2013. This worksheet calculates the NEO distance based on observed parallax and projected distance between observatories perpendicular to the line of sight to the asteroid.

Useful constants and conversion factors

$AU := 1.5 \cdot 10^8 \cdot km \qquad R_e := 6378.16 \cdot km \qquad hr := \frac{\pi}{12}$

1. Measure parallax using near-simultaneous images

NEO α,δ measured using WCS astrometric solution on images using images taken at 08:03:40 UT. N.B. Field stars α,δ agree within 0.1 arcsec.

$\delta := -22.84 \cdot deg$

$\alpha_r := 58.06 \qquad \alpha_s := 58.77 \qquad \Delta\alpha := (\alpha_s - \alpha_r) \cdot 15 \cdot \cos(\delta) \cdot arcsec$

$\delta_r := 33.9 \qquad \delta_s := 38.6 \qquad \Delta\delta := (\delta_r - \delta_s) \cdot arcsec$

$\Theta := \sqrt{(\Delta\alpha)^2 + (\Delta\delta)^2} \qquad \Theta = 10.88 \cdot arcsec$

Note: Measurement uncertainty is appx. +/-2 pixels [~ 1.6"] in each difference, so uncertainty in parallax is:

$\delta\Theta := \sqrt{2} \cdot 0.2 \cdot arcsec \qquad \delta\Theta = 0.28 \cdot arcsec$

2. Baseline length from Rigel to SSO

$\lambda_r := -110.60178 \cdot deg \qquad \varphi_r := 31.665578 \cdot deg$ Winer observatory lat/long

$\lambda_s := -119.775 \cdot deg \qquad \varphi_s := 38.811 \cdot deg$ SSO observatory lat/long

$$R_r := R_e \cdot \begin{bmatrix} \cos(\varphi_r) \cdot \cos(\lambda_r) \\ (\cos(\varphi_r) \cdot \sin(\lambda_r)) \\ \sin(\varphi_r) \end{bmatrix} \qquad R_s := R_e \cdot \begin{pmatrix} \cos(\varphi_s) \cdot \cos(\lambda_s) \\ \cos(\varphi_s) \cdot \sin(\lambda_s) \\ \sin(\varphi_s) \end{pmatrix}$$

$R := R_s - R_r$ $\qquad |R| = 1149.8 \cdot km$

This is the chord distance in xyz system with x pointing to Greenwich and z to north celestial pole

3. Projected baseline in direction of the asteroid

To calculate the projected baseline length perpendicular to the line of sight to the NEO, w first calculate the chord distance [above], then calculate the unit normal vector in directio of asteroid using same xyz coordinate system as R vector (x => direction to Greenwich, z => NCP, right-hand system), then take the magnitude of the cross product.

To calculate the unit normal vector pointing to the NEO, we need the Greenwich hour angle (GHA), which is the local hour angle (HA) plus the longitude at one of the observatories. The HA can be obtained from the FITS header.

$$\alpha := \left(8 + \frac{47}{60} + \frac{58}{3600}\right) \cdot hr \qquad \delta := -\left(22 + \frac{50}{60}\right) \cdot deg$$

Apparent NEO celestial coords at time of obs. (at SSO)

$$HA := -\left(1 + \frac{12}{60} + \frac{24}{3600}\right) \cdot hr$$

Hour angle of NEO at SSO (from FITS header)

$$GHA := HA - \lambda_s$$

Greenwich hour angle $\qquad GHA = 6.778 \cdot hr$

$$n := \begin{pmatrix} \cos(GHA) \cdot \cos(\delta) \\ -\sin(GHA) \cdot \cos(\delta) \\ \sin(\delta) \end{pmatrix}$$

Unit vector in direction of NEO $\qquad n = \begin{pmatrix} -0.187 \\ -0.903 \\ -0.388 \end{pmatrix}$

$|R \times n| = 784.3 \cdot km$

Projected baseline perpendicular to the NEO line of sight at time of observation.

3. Distance to NEO

Since the parallax triangle is so small we can find the distance as sum of topocentric and distance:

$$d_{geo} := \frac{|R \times n|}{\Theta} + R_e \qquad d_{geo} = 1.487 \times 10^{7} \cdot km \qquad d_{geo} = 0.099 \cdot AU$$

Uncertainty:

$$\delta d := \frac{|R \times n|}{\Theta - \delta\Theta} + R_e - d_{geo} \qquad \delta d = 0.0026 \cdot AU \qquad \frac{\delta d}{d_{geo}} = 2.667 \cdot \%$$

The JPL Horizons site lists the range as 0.0971 AU at the time of observation, so the fractional error is 2.1%, which is within (1σ) the measurement uncertainty of the parallax measurement.

$$d_0 := 0.0971 \cdot AU \qquad \frac{|d_0 - d_{geo}|}{d_0} = 2.1 \cdot \%$$