NEO 2012 QG42 distance from parallax

NEO 2012 QG42 was observed at Rigel and SSO observatories on Sept 16, 2012 at 04:52:00 (within 1 sec). This worksheet calculates the NEO distance based on observed parallax and projected distance between observatories perpendicular to the line of sight to the asteroid. The images are shown below.

SSO sub-image at 04:52:00.8 UT

Rigel sub-image at 04:52:00.8 UT
Useful constants and conversion factors

\[ \text{AU} := 1.5 \times 10^8 \text{km} \quad R_e := 6378.16 \text{km} \quad \text{hr} := \frac{\pi}{12} \]

1. Measure parallax using near-simultaneous images

NEO position was measured w.r.t. to a nearby star (beginning of streak) using images taken at 04:52:00.8 (SSO) and 04:52:0 (Rigel) above.

\[ \text{rpixels} := 0.725 \text{ arcsec} \quad x_r := 54 \text{ rpixels} \quad y_r := 127 \text{ rpixels} \]

\[ \text{spixels} := 0.80 \text{ arcsec} \quad x_s := 108 \text{ spixels} \quad y_s := 91 \text{ rpixels} \]

\[ \Theta := \sqrt{(x_r - x_s)^2 + (y_r - y_s)^2} \quad \Theta = 53.98 \text{ arcsec} \]

Note: Measurement uncertainty is appx. +/-2 pixels [~ 1.6"] in each difference, so uncertainty in parallax is:

\[ \delta \Theta := \sqrt{2} \times 1.6 \text{ arcsec} \quad \delta \Theta = 2.26 \text{ arcsec} \]

2. Baseline length from Rigel to SSO

\[ \lambda_r := -110.60178 \text{ deg} \quad \varphi_r := 31.665578 \text{ deg} \quad \text{Winer observatory lat/long} \]

\[ \lambda_s := -119.775 \text{ deg} \quad \varphi_s := 38.811 \text{ deg} \quad \text{SSO observatory lat/long} \]

\[ R_r := R_e \cdot \begin{pmatrix} \cos(\varphi_r) \cdot \cos(\lambda_r) \\ \cos(\varphi_r) \cdot \sin(\lambda_r) \\ \sin(\varphi_r) \end{pmatrix} \quad R_s := R_e \cdot \begin{pmatrix} \cos(\varphi_s) \cdot \cos(\lambda_s) \\ \cos(\varphi_s) \cdot \sin(\lambda_s) \\ \sin(\varphi_s) \end{pmatrix} \]

\[ R := R_r - R_s \quad |R| = 1149.8 \text{ km} \]

This is the chord distance in xyz system with x pointing to Greenwich and z to north celestial pole.
3. **Projected baseline in direction of the asteroid**

To calculate the projected baseline length perpendicular to the line of sight to the NEO, we first calculate the chord distance [above], then calculate the unit normal vector in direction of asteroid using same xyz coordinate system as R vector (x => direction to Greenwich, z => NCP, right-hand system), then take the magnitude of the cross product.

To calculate the unit normal vector pointing to the NEO, we need the Greenwich hour angle (GHA), which is the local hour angle (HA) plus the longitude at one of the observatories. The HA can be obtained from the FITS header.

\[
\alpha := \left(16 + \frac{33}{60} + \frac{20}{3600}\right)\text{hr} \quad \delta := \left(23 + \frac{15}{60}\right)\text{deg} \quad \text{Apparent NEO celestial coords at time of obs. (at SSO)}
\]

\[
HA := \left(4 + \frac{1}{60} + \frac{55}{3600}\right)\text{hr} \quad \text{Hour angle of NEO at SSO (from FITS header)}
\]

\[
GHA := HA - \lambda_s \quad \text{Greenwich hour angle} \quad GHA = 12.017\text{hr}
\]

\[
n := \begin{bmatrix}
\cos(GHA)\cdot\cos(\delta) \\
-(\sin(GHA)\cdot\cos(\delta)) \\
\sin(\delta)
\end{bmatrix} \quad \text{Unit vector in direction of NEO}
\]

\[|R \times n| = 852.1\text{\ km} \quad \text{Projected baseline perpendicular to the NEO line of sight at time of observation.}\]

3. **Distance to NEO**

Since the parallax triangle is so small we can find the distance as sum of topocentric and distance:

\[d_{geo} := \frac{|R \times n|}{\Theta} + R_{e} \quad d_{geo} = 3.262 \times 10^6\text{\ km} \quad d_{geo} = 0.0217\text{\ AU}\]

Uncertainty:

\[\delta d := \frac{|R \times n|}{\Theta - \delta \Theta} + R_{e} - d_{geo} \quad \delta d = 0.0009\text{\ AU} \quad d_{geo} + \delta d = 0.0227\text{\ AU}\]

The JPL Horizons site lists the range as 0.0228 AU at the time of observation, so the fractional error is 4.8%, which is within (1\sigma) the measurement uncertainty of the parallax measurement.