The geometry of the Universe

- How many degrees in the angles in a triangle?
- Positively/negatively curved spaces
- Measuring curvature using the cosmic microwave background
Why can't we see radiation produced during the first 1,000 years after the Big Bang?

A) It was absorbed soon after it was emitted.  
B) It hasn't reached us yet.  
C) It has been deflected by black holes.  
D) It passed by our part of the universe a few billion years ago.
Two dimensional geometry

• Only two directions: up/down and left/right
  : north/south and east/west

• All motion of particles, light confined to two dimensions

• Examples: black board, piece of paper, surface of sphere, surface of donut, surface of saddle
Geometry

• How are the diameter and circumference of a circle related?

• What is the sum of all of the angles in a triangle?
Geometry in flat space

- Circumference = $2\pi \times$ radius
  
  $\pi = 3.1415926\ldots$   $2\pi = 6.28\ldots$

- The sum of the angles in a triangle is $180^\circ$
Non-Euclidean Geometry

• How about in a curved space?

• Do demo 8C10.50
Geometry in positively curved space

- Circumference $< 2\pi \times \text{radius}$
- The sum of the angles in a triangle $> 180^\circ$
Properties of positively curved space

- Finite
- Unbounded
- No center

How do the properties of positively curved space differ from flat space?
Geometry in negatively curved space

- Circumference $> 2\pi \times \text{radius}$
- The sum of the angles in a triangle $< 180^\circ$
Curvature of the Universe

The curvature of the Universe is determined by:

- the density of matter and energy
  - higher density produces positive curvature
  - gravity from matter always makes positive curvature
- the expansion of the Universe
  - more rapid expansion produces negative curvature

At the “critical density”, expansion exactly balances gravity – universe is flat
Fate of the Universe

Strong throw: $v \geq v_{\text{esc}}$

Maximum height

Weak throw: $v < v_{\text{esc}}$

Asteroid
Fate of the Universe

- Negatively curved space
- Flat space
- Positively curved space

$\Omega_0 < 1$
$\Omega_0 = 1$
$\Omega_0 > 1$
Curvature of the Universe

The curvature of the Universe is determined by the density parameter $\Omega_0$

$$\Omega_0 = \frac{\rho}{\rho_c}$$

$\rho$ = density of matter/energy
Critical density $\rho_c = 10^{-26}$ kg/m³

$\Omega_0 < 1 \Rightarrow$ negative curvature

$\Omega_0 > 1 \Rightarrow$ positive curvature
Cosmic Microwave Background

Small fluctuations are due to sound waves at recombination.
Temperature variations in the Cosmic Microwave Background (CMB) are observed to be about 0.0003 K. The expected physical size of the hot/cold regions can be calculated.
Curvature of the Universe

The curvature of the Universe is determined by the density parameter $\Omega_0$

$$\Omega_0 = \frac{\rho}{\rho_c}$$

$\Omega_0 < 1 \Rightarrow$ negative curvature

$\Omega_0 > 1 \Rightarrow$ positive curvature

Measurement of CMB fluctuations gives

$$\Omega_0 = 1.02 \pm 0.02$$